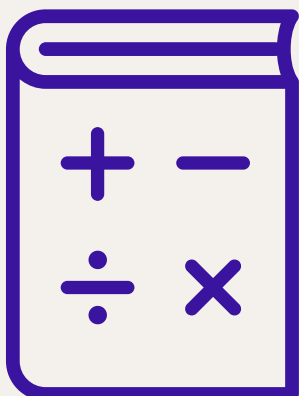
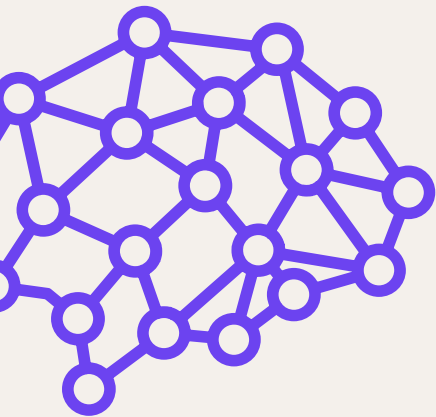




Improving mathematics in upper primary and lower secondary



This Guidance Report is based on original content from a report titled *Improving Mathematics in Key Stage Two and Three* produced by the Education Endowment Foundation (EEF). The original content has been modified where appropriate for Australian context.

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Evidence for Learning (E4L) thanks the Australian researchers and practitioners who provided input to and feedback on drafts of this Guidance Report and in particular recognise the considered review by the Australian Association of Mathematics Teachers (AAMT).

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Foreword

Leaving school with good results in maths is a prerequisite for progressing into quality jobs, apprenticeships, and further education. The skills we learn at school help us with everyday life too, whether that's putting up a shelf or working out how much money we have left after we've paid our bills. Yet too many of our young people do not make the grade and, as a result, risk social and economic exclusion.

These students come disproportionately from disadvantaged homes. A 15-year-old from a low socio-economic background is nearly four times more likely to be below expected levels for maths achievement than classmates from more advantaged homes.¹

To truly break this link between family income and educational attainment, we have to start early and make sure that all young people – regardless of background – have access to great maths teaching in primary and secondary school.

At Evidence for Learning, we believe the best way to do this is through better use of evidence: looking at what has – and has not – worked in the past can put us in a much better place to judge what is likely to work in the future. But it can be difficult to know where to start. There are thousands of studies of maths teaching, most of which are presented in academic papers and journals. Teachers are inundated with information about programs and training courses too, all of which make claims about impact. How can anyone know which findings are the most secure, reliable, and relevant to their school and students?

This is why we have produced this Guidance Report. Developed by our UK partner, the Education Endowment Foundation (EEF) and updated for Australian audiences, it offers eight practical, evidence-based recommendations – that are relevant to all students – but particularly to those struggling with their maths. To develop the recommendations, the EEF reviewed the best available international research and consulted experts, teachers, and academics to arrive at key principles for effective teaching.

Of course, this guide on its own will not create better maths learners. It is only when the research knowledge summarised in this guide is combined with teachers' professional judgement and expertise that students in classrooms will benefit.

We hope you will appreciate our contribution to the shared endeavour of consistently excellent maths development in schools.

The Evidence for Learning Team



Introduction

What is this guide for?

This Guidance Report focuses on the teaching of maths to students in upper primary and lower secondary school (ages 7 to 14). The decision to focus on this age range was made after an initial consultation period with teachers, academics, and other stakeholders. The consultation suggested that these were areas where guidance could make a big impact as not only are schools seeking advice on adjusting to a new curriculum, there is also concern about students making a transition between primary and secondary school.

This report is not intended to provide a comprehensive guide to maths teaching. We have made recommendations where there are research findings that schools can use to make a significant difference to students' learning, and have focused on the questions that appear to be most salient to practitioners. There are aspects of maths teaching not covered by this guide. In these situations, teachers must draw on their knowledge of maths, professional experience and judgement, and assessment of their students' knowledge and understanding.

The focus is on improving the quality of teaching. Excellent maths teaching requires good content knowledge, but this is not sufficient. Excellent teachers also know the ways in which students learn maths and the difficulties they are likely to encounter, and how maths can be most effectively taught.^{2,3,4}

The guide draws on a review of the evidence on effective maths teaching. As such, it is not a new study in itself, but rather is intended as an accessible overview of existing research with clear, actionable guidance. More information about how this guide was created is available at the end of the report.

Who is this guide for?

This guide is aimed primarily at subject leaders, principals, and other staff with responsibility for leading improvements in maths teaching in primary and secondary schools. Classroom teachers and Teaching Assistants will also find this guide useful as a resource to aid their day-to-day teaching. It may also be used by:

- school councils and parents to support and challenge school staff;
- program developers to inform their development of both professional learning for teachers and interventions for students; and
- educational researchers to conduct further testing of the recommendations in this guide, and fill in gaps in the evidence.

Using this guide

We recognise that the effective implementation of these recommendations – such that they make a real impact on students – is both critical and challenging. There are several key principles to consider when acting on this guide.

1. Professional Learning (PL) will be an important component of implementation and is key to raising the quality of teaching and teacher knowledge.
2. These recommendations do not provide a 'one size fits all' solution. It is important to consider the delicate balance between implementing the recommendations faithfully and applying them appropriately in a school's particular context. Implementing the recommendations effectively will therefore require careful consideration of context as well as sound professional judgement.

3. It is important to consider the precise detail provided beneath the headline recommendations. For example, schools should not use Recommendation 7 to justify the purchase of lots of interventions. Rather, it should provoke thought about the most appropriate interventions to buy.
4. Inevitably, change takes time, and we recommend taking at least two terms to plan, develop, and pilot strategies on a small scale before rolling out new practices across the school. Gather support for change across the school and set aside regular time throughout the year to focus on this project and review progress.

The section 'Acting on the evidence', suggests a range of strategies and tools that you might find helpful in planning, structuring and delivering an approach to improving maths.



Summary of recommendations

1



Use assessment to build on students' existing knowledge and understanding

- Assessment should be used not only to track students' learning but also to provide teachers with information about what students do and do not know
- This should inform the planning of future lessons and the focus of targeted support
- Effective feedback will be an important element of teachers' response to assessment
- Feedback should be specific and clear, encourage and support further effort, and be given sparingly
- Teachers not only have to address misconceptions but also understand why students may persist with errors
- Knowledge of common misconceptions can be invaluable in planning lessons to address errors before they arise

See page 8

2



Use manipulatives and representations

- Manipulatives (physical objects used to teach maths) and representations (such as number lines and graphs) can help students engage with mathematical ideas
- However, manipulatives and representations are just tools: how they are used is essential
- They need to be used purposefully and appropriately to have an impact
- There must be a clear rationale for using a particular manipulative or representation to teach a specific mathematical concept
- Manipulatives should be temporary; they should act as a 'scaffold' that can be removed once independence is achieved

See page 11

3



Teach strategies for solving problems

- If students lack a well-rehearsed and readily available method to solve a problem they need to draw on problem-solving strategies to make sense of the unfamiliar situation
- Select problem-solving tasks for which students do not have ready-made solutions
- Teach students to use and compare different approaches
- Show students how to interrogate and use their existing knowledge to solve problems
- Use worked examples to enable students to analyse the use of different strategies
- Require students to monitor, reflect on, and communicate their problem solving

See page 15

4



Enable students to develop a rich network of mathematical knowledge

- Emphasise the many connections between mathematical facts, procedures, and concepts
- Ensure that students develop fluent recall of facts
- Teach students to understand procedures
- Teach students to consciously choose between mathematical strategies
- Build on students' informal understanding of sharing and proportionality to introduce procedures
- Teach students that fractions and decimals extend the number system beyond whole numbers
- Teach students to recognise and use mathematical structure

See page 18

5



Develop students' independence and motivation

- Encourage students to take responsibility for, and play an active role in, their own learning
- This requires students to develop metacognition – the ability to independently plan, monitor and evaluate their thinking and learning
- Initially, teachers may have to model metacognition by describing their own thinking
- Provide regular opportunities for students to develop metacognition by encouraging them to explain their thinking to themselves and others
- Avoid doing too much too early
- Positive attitudes are important, but there is scant evidence on the most effective ways to foster them
- School leaders should ensure that all staff, including non-teaching staff, encourage enjoyment in maths for all children

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21

6



Use tasks and resources to challenge and support students' mathematics

- Tasks and resources are just tools – they will not be effective if they are used inappropriately by the teacher
- Use assessment of students' strengths and weaknesses to inform your choice of task
- Use tasks to address student misconceptions
- Provide examples and non-examples of concepts
- Use stories and problems to help students understand mathematics
- Use tasks to build conceptual knowledge in tandem with procedural knowledge
- Technology is not a silver bullet – it has to be used judiciously and less costly resources may be just as effective

See page
24

7



Use structured interventions to provide additional support

- Selection should be guided by student assessment
- Interventions should start early, be evidence-based and be carefully planned
- Interventions should include explicit and systematic instruction
- Even the best-designed intervention will not work if implementation is poor
- Support students to understand how interventions are connected to whole-class instruction
- Interventions should motivate students – not bore them or cause them to be anxious
- If interventions cause students to miss activities they enjoy, or content they need to learn, teachers should ask if the interventions are really necessary
- Avoid 'intervention fatigue'. Interventions do not always need to be time-consuming or intensive to be effective

See page
31

8



Support students to make a successful transition between primary and secondary school

- There is a large dip in mathematical attainment and attitudes towards maths as children move from primary to secondary school
- Primary and secondary schools should develop shared understandings of curriculum, teaching and learning
- When students arrive in Year 7, quickly attain a good understanding of their strengths and weaknesses
- Structured intervention support may be required for Year 7 students who are struggling to make progress
- Carefully consider how students are allocated to maths classes
- Setting is likely to lead to a widening of the attainment gap between disadvantaged students and their peers, because the former are more likely to be assigned to lower groups

See page
34

1

Use assessment to
build on students'
existing knowledge
and understanding



1

Use assessment to build on students' existing knowledge and understanding



Mathematical knowledge and understanding can be thought of as consisting of several components and it is quite possible for students to have strengths in one component and weaknesses in another.⁵ It is therefore important that assessment is not just used to track students' learning but also provides teachers with up-to-date and accurate information about the specifics of what students do and do not know. This information allows teachers to adapt their teaching so it builds on students' existing knowledge, addresses their weaknesses, and focuses on the next steps that they need in order to make progress. Formal tests can be useful here, although assessment can also be based on evidence from low-stakes class assessments, informal observation of students, or discussions with them about maths. More guidance on how to conduct useful and accurate assessment is available in our UK partner, the EEF's guidance on [Assessing and Monitoring Pupil Progress](#).

Responding to assessment

Teachers' knowledge of students' strengths and weaknesses should inform the planning of future lessons and the focus of targeted support (see [Recommendation 7](#)). Teachers may also need to try a different approach if it appears that what they tried the first time did not work. Effective feedback will be an important element of teachers' responses to assessment information. Consider the following characteristics of effective feedback:⁶

- **be specific, accurate, and clear** (for example, 'You are now factorising numbers efficiently, by taking out larger factors earlier on', rather than, 'Your factorising is getting better');
- **give feedback sparingly so that it is meaningful** (for example, 'One of the angles you calculated in this problem is incorrect – can you find which one and correct it?');
- **compare what a student is doing right now with what they have done wrong before** (for example, 'Your rounding of your final answers is much more accurate than it used to be');

- **encourage and support further effort** by helping students identify things that are hard and require extra attention (for example, 'You need to put extra effort into checking that your final answer makes sense and is a reasonable size');
- **provide guidance to students on how to respond to teachers' comments**, and give them time to do so; and
- **provide specific guidance on how to improve** rather than just telling students when they are incorrect (for example, 'When you are unsure about adding and subtracting numbers, try placing them on a number line', rather than 'Your answer should be -3 not 3').

Feedback needs to be efficient. Schools should be careful that their desire to provide effective feedback does not lead to onerous marking policies and a heavy teacher workload.⁷ Effective feedback can be given orally; it doesn't have to be in the form of written marking.

Evidence summary

Feedback is generally found to have large effects on learning. The review identified two meta-analyses indicating that the effects of feedback in maths are similar to other subjects. There is considerable variability in reported effects, and, feedback can have powerful negative as well as positive impacts on learning.

There is detailed literature on the [misconceptions](#) that students often develop when learning maths.



Addressing misconceptions

A misconception is an understanding that leads to a 'systematic pattern of errors'.⁸ Often misconceptions are formed when knowledge has been applied outside of the context in which it is useful. For example, the 'multiplication makes bigger, division makes smaller' conception applies to positive, whole numbers greater than 1. However, when subsequent mathematical concepts appear (for example, numbers less than or equal to 1), this conception, extended beyond its useful context, becomes a misconception.^{9,10,11}

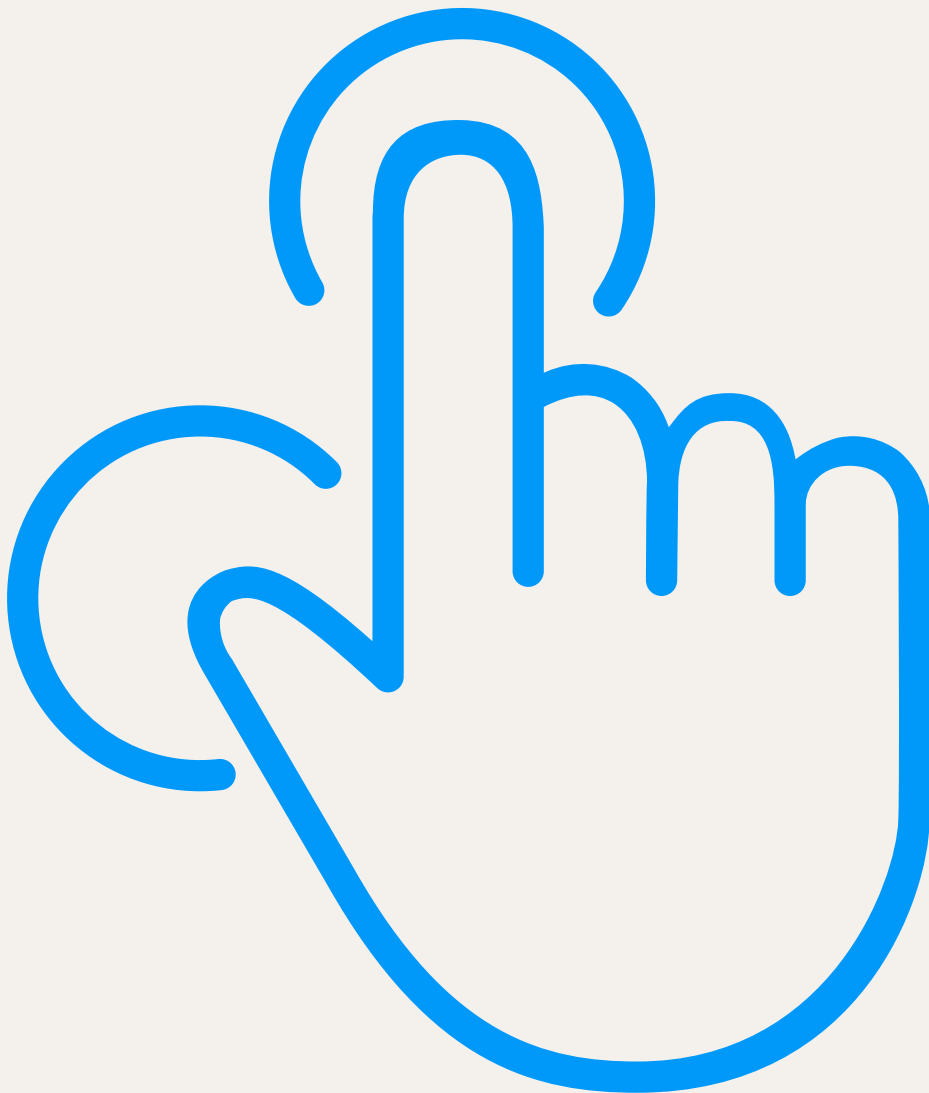
It is important that misconceptions are uncovered and addressed rather than side-stepped or ignored. Students will often defend their misconceptions, especially if they are based on sound, albeit limited, ideas. In this situation, teachers could think about how a misconception might have arisen and explore with students the 'partial truth' that it is built on and the circumstances where it no longer applies.¹¹ Counter-examples can be effective in challenging students' belief in a misconception. However, students may need time and teacher support to develop richer and more robust conceptions.

Knowledge of the common errors and misconceptions in maths can be invaluable when designing and responding to assessment, as well as for predicting the difficulties learners are likely to encounter in advance.⁹ Teachers with knowledge of the common misconceptions can plan lessons to address potential misconceptions before they arise, for example, by comparing examples to non-examples when teaching new concepts. A non-example is something that is not an example of the concept.



2

Use manipulatives
and representations





Manipulatives and representations can be powerful tools for supporting students to engage with mathematical ideas. However, manipulatives and representations are just tools: *how* they are used is important. They need to be used purposefully and appropriately in order to have an impact. Teachers should ensure that there is a clear rationale for using a particular manipulative or representation to teach a specific mathematical concept. The aim is to use manipulatives and representations to reveal mathematical structures and enable students to understand and use maths independently.

Box A: What are manipulatives and representations?

A manipulative is a physical object that students or teachers can touch and move which is used to support the teaching and learning of maths. Common manipulatives include Cuisenaire rods and Base Ten blocks or multibase arithmetic blocks.

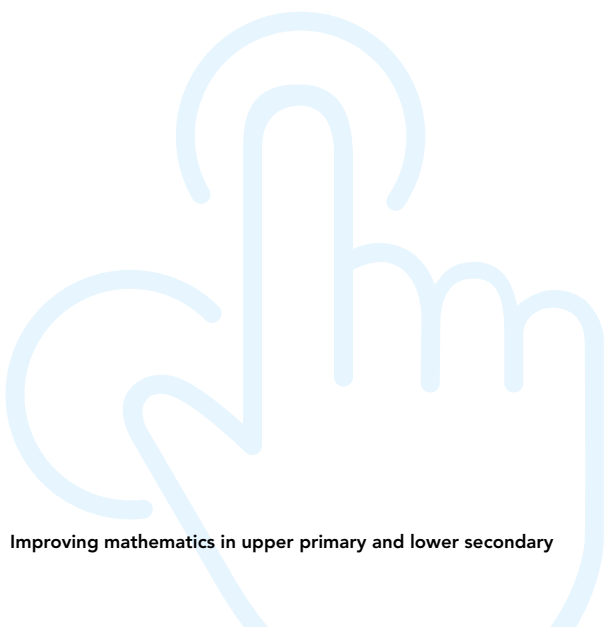
The term 'representation' refers to a particular form in which maths is presented.¹² Examples of different representations include:

- two fractions represented on a number line;
- a quadratic function expressed algebraically or presented visually as a graph; and
- a probability distribution presented in a table or represented as a histogram.

What does effective use of manipulatives look like?

Manipulatives can be used across all ages covered in this report. The evidence suggests some key considerations:

- **Ensure that there is a clear rationale** for using a particular manipulative or representation to teach a specific mathematical concept. Manipulatives should be used to provide insights into increasingly sophisticated maths.
- **Enable students to understand the links between the manipulatives and the mathematical ideas they represent.**¹³ This requires teachers to encourage students to link the materials (and the actions performed on or with them) to the maths of the situation, to appreciate the limitations of concrete materials, and to develop related mathematical images, representations, and symbols.¹⁴
- **Try to avoid students becoming reliant on manipulatives** to do a type of task or question. A manipulative should enable a student to understand maths by illuminating the underlying general relationships, not just 'getting them to the right answer' to a specific problem.¹³
- **Manipulatives should act as a 'scaffold', which can be removed** once independence is achieved. Before using a manipulative, it is important to consider how it can enable students to eventually do the maths *without* it. When moving away from manipulatives, students may find it helpful to draw diagrams or imagine using the manipulatives.
- **Manipulatives can be used to support students of all ages.** The decision to remove a manipulative should be made in response to the students' improved knowledge and understanding, not their age.





Box B: Using manipulatives – an example

A teacher said, 'Give me a two digit number ending in 0.' A student said, 'Forty.'

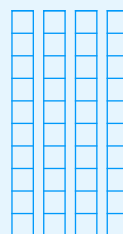
The teacher said, 'I'm going to subtract the tens digit from the number: $40 - 4$ gives me 36.'

The students tried this with other two-digit numbers ending in 0 and discovered that the result was always a multiple of 9.

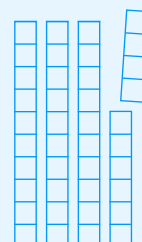
The teacher said, 'I'm going to use multilink cubes to see whether this will help us see why we always get a multiple of 9.'

The teacher made four sticks of 10 cubes each

'So here is 40. What does it look like if we remove 4 cubes?'

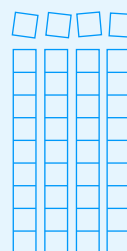


A student came to the front of the classroom and removed 4 cubes.



'How else could you do it?' asked the teacher.

Another student removed 4 cubes in a different way.



A student said, 'Ah yes! If we take away one from each 10 then we are left with four 9s.'

Another student said, 'And if we started with 70 we'd have 10 sevens take away 1 seven is 9 sevens.'

The teacher wrote:

$$40 - 4 = 10 \times 4 - 1 \times 4 = 9 \times 4$$

$$70 - 7 = 10 \times 7 - 1 \times 7 = 9 \times 7$$

and the students discussed what is going on here, before the teacher concluded with the generalisation:

$$10t - t = (10 - 1)t = 9t$$



What about other types of representation?

The evidence indicates that number lines are a particularly effective representation for teaching across all ages covered in this Guidance Report and that there is strong evidence to support the use of diagrams as a problem-solving strategy. The specific evidence regarding the use of representations more generally is weaker, however, it is likely that the points above regarding effective use of manipulatives apply to all other representations.

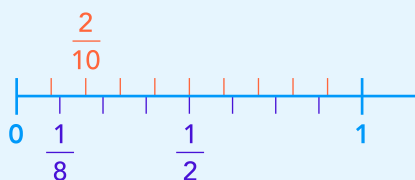
Box C: Using a number line

The teacher noticed that some students were incorrectly adding fractions by adding the numerators and the denominators. She gave the class this task:

Is this true?

$$\frac{1}{2} + \frac{1}{8} = \frac{2}{10}$$

Some students noticed that $\frac{2}{10}$ is less than $\frac{1}{2}$. With the teacher's help, the students represented the three fractions using a number line.



This helped students to see that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$, and then to work out that the answer is $\frac{5}{8}$.

The students then invented their own examples of incorrect and correct fraction additions using number lines to make sense of it.

While in general the use of multiple representations appears to have a positive impact on attainment, more research is needed to inform teachers' choices about which, and how many, representations to use when.¹⁵ There is promising evidence that comparison and discussion of different representations can help students develop conceptual understanding. Teachers should purposefully select different representations of key mathematical ideas to discuss and compare with the aim of supporting students to develop more abstract, diagrammatic representations. However, while using multiple representations can aid understanding, teachers should be aware that using too many representations at one time may cause confusion and hinder learning.¹⁵

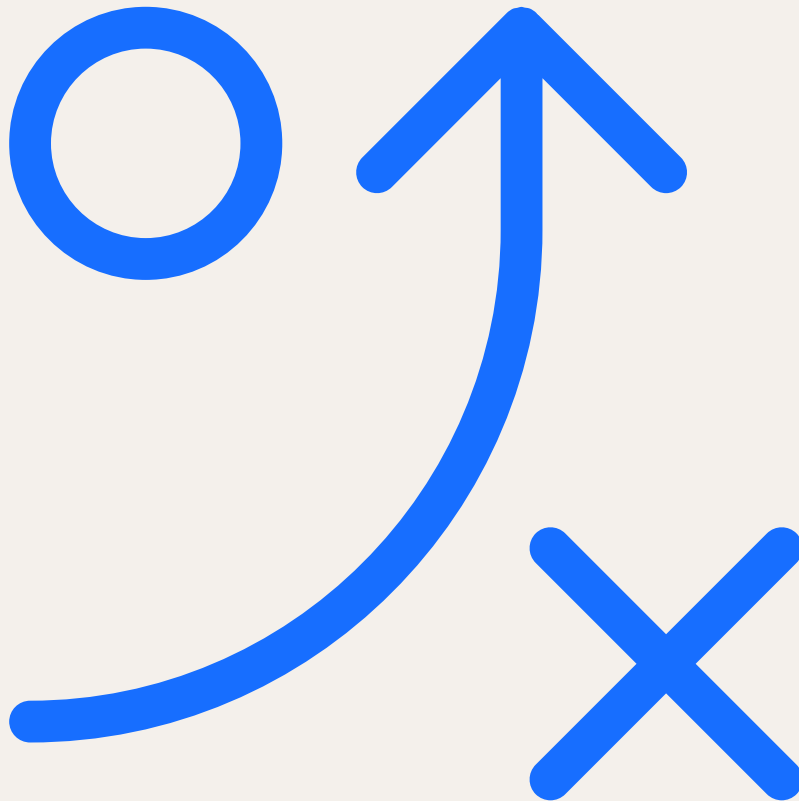
Evidence summary

The review identified five relevant meta-analyses concerned with the use of concrete manipulatives and representations. The evidence was stronger in support of concrete manipulatives.

Two systematic reviews by the US What Works Clearinghouse provide evidence to support the use of visual representations, particularly in problem solving and to support students who are struggling with maths.

3

Teach strategies for
solving problems





Problem solving generally refers to situations in which students do not have a readily-available method that they can use. Instead, they have to approach the problem flexibly and work out a solution for themselves. To succeed in this, students need to draw on a variety of *problem-solving strategies* (see Box D) which enable them to make sense of unfamiliar situations and tackle them intelligently.

Box D: What is a problem-solving strategy?

A problem-solving strategy is a general approach to solving a problem. The same general strategy can be applied to solving a variety of different problems. For example, a useful problem-solving strategy is to identify a *simpler* but related problem. Discussing the solution to the simpler problem can give insight into how the original, harder problem may be tackled and the underlying mathematical structure. A strategy is different from an algorithm, which is a well-established sequence of predetermined steps that are executed in a particular order to carry out a commonly-required procedure.

The evidence suggests that teachers should consider the following when developing these skills.¹⁶

- **Select genuine problem-solving tasks that students do not have well-rehearsed, ready-made methods to solve.** Sometimes problem-solving is taken to mean routine questions set in context, or 'word problems', designed to illustrate the use of a specific method. But if students are only required to carry out a given procedure or algorithm to arrive at the solution, it is not really problem solving; rather, it is just practising the procedure.
- **Consider organising teaching so that problems with *similar structures* and *different contexts* are presented together, and, likewise, that problems with the *same context* but *different structures* are presented together.** Students need to experience identifying similar maths that underlies different situations, and also to identify and interrogate multiple relationships between variables in one situation.
- **Teach students to use and compare different approaches.** There are often multiple ways to approach a problem. Much can be learned by examining different solutions to the same problem and looking for similarities in solution approaches to different problems. Students will need to distinguish between superficial similarity (for example, two problems both about carrots) and deeper similarities, relating to mathematical structure, which make similar strategies effective (such as two problems in different contexts that are both about enlargement).
- **Teach students to interrogate and use their existing mathematical knowledge to solve problems.** Students should be encouraged to search their knowledge of similar problems they have encountered for strategies that were successful, and for facts and concepts that might be relevant.
- **Encourage students to use visual representations.** Help students to make use of appropriate diagrams and representations that provide insight into the structure of a problem and into its mathematical formulation.
- **Use worked examples to enable students to analyse the use of different strategies.** Worked examples, or 'solved problems', present the problem and a correct solution together; they remove the need to carry out the procedures required to reach the solution and enable students to focus on the reasoning and strategies involved. Worked examples may be complete, incomplete, or incorrect, deliberately containing common errors and misconceptions for learners to uncover. Analysing and discussing worked examples helps students develop a deeper understanding of the logical processes used to solve problems.



- **Require students to monitor, reflect on, and communicate their reasoning and choice of strategy.** While working on a problem, encourage students to ask questions like, 'What am I trying to work out?', 'How am I going about it?', 'Is the approach that I'm taking working?', and 'What other approaches could I try?' When the problem is completed, encourage students to ask questions like, 'What worked well when solving this problem?', 'What didn't work well?', 'What other problems could be solved by a similar approach?', 'What similar problems to this one have I solved in the past?' Students should communicate their thinking verbally and in writing – using representations, expressions, and equations – to both teachers and other students. Evidence for Learning's Guidance Report *Metacognition and self-regulated learning* provides further guidance on supporting students to monitor their own learning.

Evidence summary

The review identified three relevant meta-analyses concerned with the teaching of problem-solving. These provide some evidence for the use of problem-solving, although the effects were varied.

A systematic review by the US What Works Clearinghouse provides evidence for specific teaching approaches. The evidence strength is judged to be strongest in support of the use of visual representations and worked examples, and encouraging students to monitor and reflect on the problem-solving process.

Box E: Using the bar model to compare strategies

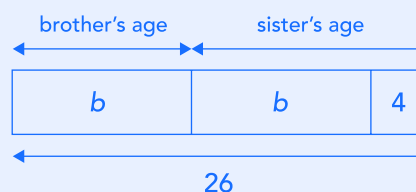
A class was working on this problem:

A sister is 4 years older than her brother.

The total of their ages is 26.

How old are they?

One student used the bar model to solve the problem:



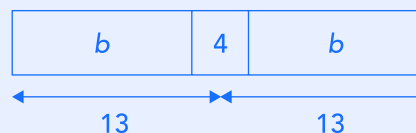
$$b + b + 4 = 26$$
$$\text{so } b + b = 22$$

so the brother is 11 years old and the sister is 15 years old.

Another student said, 'Couldn't we just halve 26 and then add and take away 4? So their ages are $13 + 4$ and $13 - 4$, which is 17 and 9.'

Another student said, 'No, you don't add and take away 4, you add and take away 2.'

The teacher drew this revised bar model diagram:

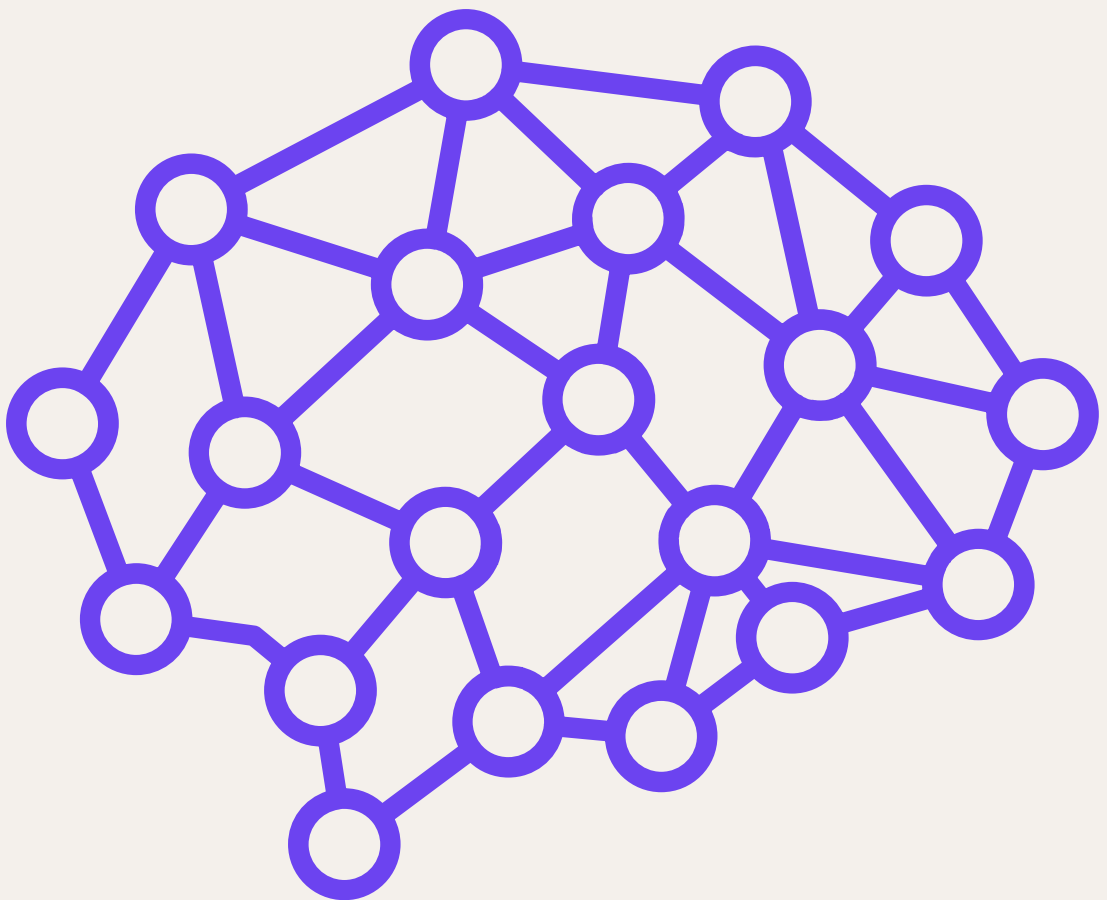


'How does this help us to see whether it's 4 or 2 that you add and subtract?'

A student said, 'There are two lots of b in the 26, there is only one 4, so it was correct to add and subtract 2, not 4.'

4

Enable students to
develop a rich network of
mathematical knowledge





This recommendation presents the evidence regarding teaching specific topics in maths. Although this recommendation concerns particular topics, teaching should emphasise the many connections between different mathematical facts, procedures, and concepts to create a rich network.

Currently, the evidence about effective teaching approaches is stronger regarding number (including fractions, ratio and proportion) and algebra than for other areas such as geometry. However, it is likely that some of the approaches below (particularly choosing between strategies, paying attention to mathematical structure, and building on students' informal knowledge) apply across mathematical topics. Teachers should adopt such approaches while drawing on their knowledge of maths, their own professional experience, and the other recommendations in this Guidance Report.

Ensure that students develop fluent recall of number facts

Quick retrieval of number facts is important for success in maths.¹⁷ It is likely that students who have problems retrieving addition, subtraction, multiplication, and division facts, including number bonds and multiples, will have difficulty understanding and using mathematical concepts they encounter later on in their studies.

Teach students to understand procedures

Students are able to apply procedures most effectively when they understand how the procedures work and in what circumstances they are useful.¹⁸ Fluent recall of a procedure is important, but teachers should ensure that appropriate time is spent on developing understanding. One reason for encouraging understanding is to enable students to reconstruct steps in a procedure that they may have forgotten.¹⁹ The recommendations in this guide on visual representations, misconceptions, and setting problems in real-world contexts are useful here.

Teach students to choose between mathematical strategies

Teachers should help students to compare and choose between different methods and strategies for solving problems in algebra, number, and elsewhere. Students should be taught a range of mental, calculator, and pencil-and-paper methods, and encouraged to consider when different methods are appropriate and efficient.

The evidence suggests that using a calculator does not generally harm students' mental or pencil-and-paper calculation skills. In fact, studies have shown using a calculator can have positive impacts, not only on mental calculation skills, but also on problem-solving and attitudes towards maths.^{20,21} Calculators should be integrated into the teaching of mental and other calculation approaches, and students should be taught to make considered decisions about when, where, and why to use particular methods. The aim is to enable students to self-regulate their use of calculators, consequently making less (but better) use of them.²²

Evidence summary

The review identified two relevant meta-analyses concerning the teaching of algebra. However, these meta-analyses are at a general level and do not provide evidence about specific teaching approaches.

Four systematic reviews by the US What Works Clearinghouse provide evidence for specific teaching approaches in number and algebra. The evidence strength is judged to be stronger in support of focusing on fluent recall, encouraging the deliberate choice of strategies and using number lines to represent fractions and decimals.

The review identified four meta-analyses investigating calculator use. The evidence on calculator use is judged to be strong.



Build on students' informal understanding of multiplicative reasoning to introduce procedures

Multiplicative reasoning is the ability to understand and think about multiplication and division. It is an important skill which is required for tasks that involve ratios, rates, and proportions, and is often required in real-life contexts such as 'best buy' problems. For example, a common misconception – when asked to work out the quantities required for 10 people from a recipe for 4 people – involves many students adding 6 (because the difference in the number of people is 6) rather than multiplying by $2\frac{1}{2}$, or doubling and adding half as much again (because the ratio is 4:10, 2:5, or $1:2\frac{1}{2}$).

There is some evidence to suggest that delaying the teaching of formal methods in order to focus on developing students' multiplicative reasoning is beneficial.¹⁸ Teachers should consider using a series of contexts and tasks which progressively build on students' informal understanding. After introducing formal procedures and algorithms, teachers should return to students' initial informal strategies and show when they lead to the same answers, as well as when and why they may be less effective.

Teach students that fractions and decimals extend the number system beyond whole numbers

Fractions are often introduced to students with the idea that they represent parts of a whole – for example, one half is one part of a whole that has two equal parts. This is an important concept, but does not extend easily to mixed fractions that are greater than 1. Another important concept is often overlooked: fractions are numbers which can be represented on the number line. They have magnitudes or values, and they can be used to refer to numbers in-between whole numbers.¹⁸

Understanding that fractions are numbers, and being able to estimate where they would occur on a number line, can help students to estimate the result of adding two fractions and so recognise, and address, misconceptions such as the common error of adding fractions by adding the numerators and then adding the denominators.

Number lines are a useful tool for teaching these concepts. They can be used to:

- represent the magnitude, or value, of fractions, decimals, and rational numbers generally; and
- compare the magnitudes of fractions, decimals, and whole numbers.

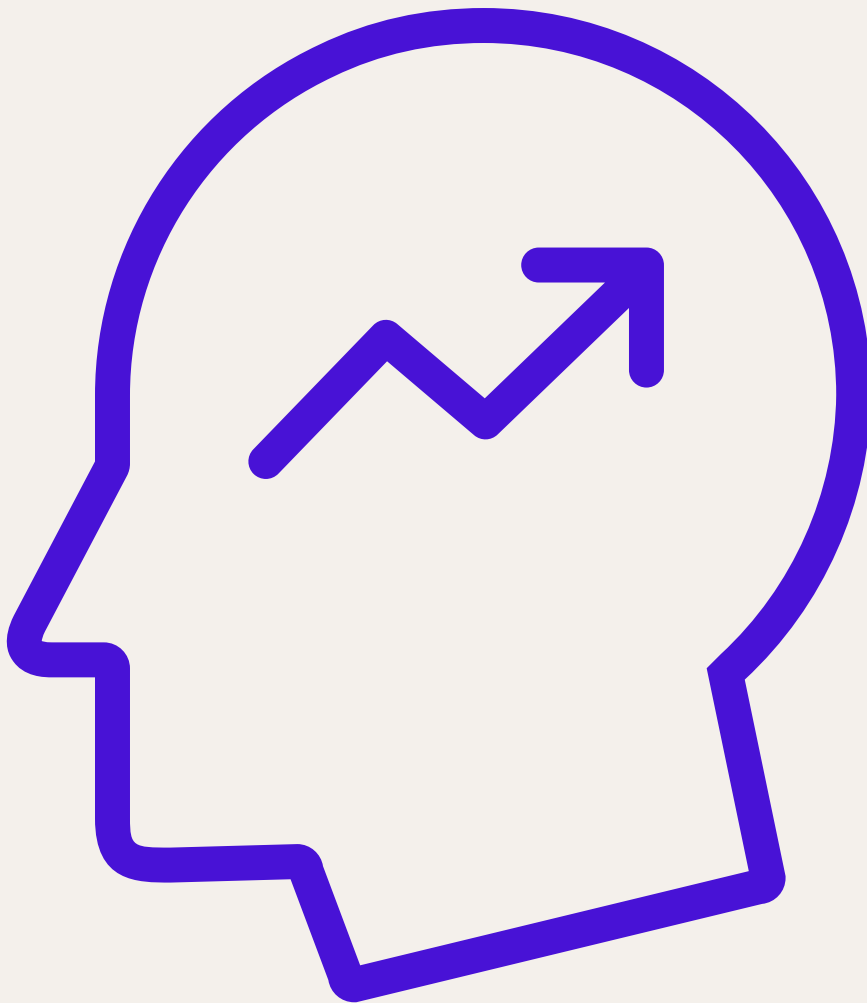
Teach students to recognise and use mathematical structure

Paying attention to underlying mathematical structure helps students make connections between problems, solution strategies, and representations that may, on the surface, appear different, but are actually mathematically equivalent. Teachers should support students to use language that reflects mathematical structure, for example by rephrasing students' responses that use vague, non-mathematical language with appropriate mathematical language. Some examples of teachers supporting students to recognise mathematical structure are:

- encouraging students to read numerical and algebraic expressions as descriptions of relationships, rather than simply as instructions to calculate¹⁴ (for example, students often regard the equals sign as an instruction to calculate rather than an indication of an equivalence;²³ understanding a relationship as an equivalence would mean thinking of ' $17 \times 25 = 10 \times 25 + 7 \times 25$ ' as ' 17×25 is the same as $10 \times 25 + 7 \times 25$ '); and
- enabling students to understand the inverse relations between addition and subtraction, and between multiplication and division.¹⁴

5

Develop students'
independence
and motivation





Teachers should encourage students to take responsibility for, and play an active role in, their own learning.²⁴ This will require students to develop metacognition (the ability to independently plan, monitor, and evaluate their thinking and learning) and motivation towards learning maths.

Evidence summary

The review identified six relevant meta-analyses concerned with approaches focused on metacognition and/or self-regulation, which provide moderate evidence for these approaches.

This recommendation is also informed by an evidence review conducted for the Guidance Report on metacognition and self-regulation.

The review identified one relevant meta-analysis concerned with worked examples, providing some weak evidence to support the use of worked examples.

Ultimately the aim is for students to be able to do this automatically and independently, without needing support from the teacher or their peers, however, these are complex skills which will initially require explicit teaching and support. Teachers should model metacognition (see example in Box F) by simultaneously describing their own thinking or asking questions of their students as they complete a task.²⁵ Worked examples could be usefully employed by the teacher to make their thinking explicit.²⁶ Teachers should carefully increase their expectations regarding students' independence as the students gain competence and fluency. Teachers can provide regular opportunities for students to develop independent metacognition through:

- encouraging self-explanation – students explaining to themselves how they planned, monitored, and evaluated their completion of a task; and
- encouraging students to explain their metacognitive thinking to the teacher and other students.²⁷

Develop students' metacognition through structured reflection on their learning

Developing metacognition – often thought of as students' ability to think about their own thinking and learning – can help them to become more effective and independent mathematicians. Examples of this ability include:

- examining existing knowledge to inform the selection of a particular approach to solving a mathematical task;
- monitoring whether the chosen approach has been successful; and then
- deliberately changing or continuing the approach based on that evidence.

Box F: Modelling metacognition during problem solving

While demonstrating the solving of a problem, a teacher could model how to plan, monitor, and evaluate their thinking by reflecting aloud on a series of questions. These could include:

- What is this problem asking?
- Have I ever seen a mathematical problem like this before? What approaches to solving it did I try and were they successful?
- Could I represent the problem with a diagram or graph?
- Does my answer make sense when I re-read the problem?
- Do I need help or more information to solve this problem? Where could I find this?



Developing metacognition is not straightforward and there are some important challenges to consider.

- Teachers need to ensure that students' metacognition does not detract from concentration on the mathematical task itself.²⁸ This might happen if students are expected to do too much, too early, without effective scaffolding from their teacher.
- Regardless of the strategy being taught, students need significant time to imitate, internalise, and independently apply strategies, with strategies used repeatedly across many maths lessons.²⁹ It is likely that the time required to develop metacognition is much greater than for other skills and knowledge.
- Discussion and dialogue can be useful tools for developing metacognition, but students may need to be taught how to engage in discussion.³⁰ Teachers should model effective discussion and 'what to do as a listener'.³¹ Orchestrating productive discussions requires considerable skill and so may require targeted professional development.

Build students' long-term motivation towards learning and doing maths

The development of positive attitudes and motivation is, of course, itself an important goal for teaching. It can also support the development of self-regulation and metacognition, as these capabilities require deliberate and sustained effort, which can require motivation over a long period of time. Motivation is complex and may be influenced by a like or dislike of maths, beliefs about whether one is good or bad at maths, and beliefs about whether maths is useful or not.³² In Australia, students attitudes towards maths tend to worsen as they get older. During the 2015 Trends in International Mathematics and Science Study (TIMSS), the proportion of Australian students who reported that they did not like learning maths at Year 4 is 27%, which at Year 8 had increased to 50% percent.³³

Although positive attitudes are important, there is a lack of evidence regarding effective approaches to developing them. It is likely to be important to model positive attitudes towards maths throughout the whole school. School leaders should ensure that all staff, including non-teaching staff, encourage and model motivation, confidence, and enjoyment in maths for all children.

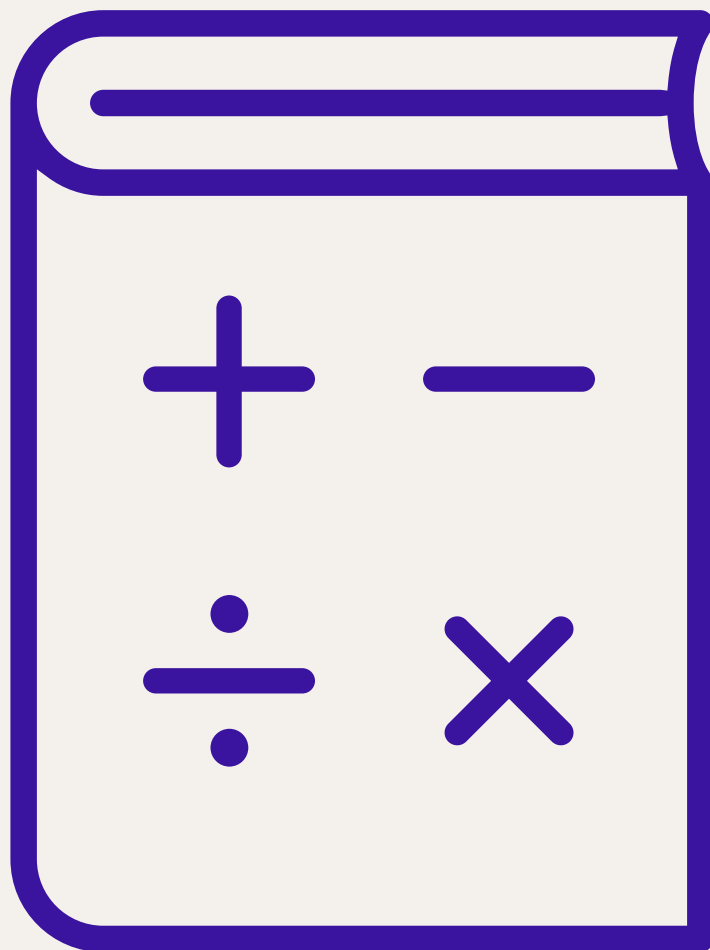
Teachers should engage parents to encourage their children to value, and develop confidence in, maths.³⁴ However, teachers should exercise caution when engaging parents directly in students' maths learning, for example by helping with homework, as interventions designed to do this have often not been linked to increased attainment.³⁵ Evidence for Learning's Guidance Report titled *Working with parents to support student's learning* provides further advice around engaging parents.

Box G: Maths anxiety

Maths anxiety is a type of anxiety that specifically interferes with maths, and is not the same as general anxiety. It can have a large detrimental impact on students' learning by overloading their working memory or causing them to avoid maths. Maths anxiety tends to increase with age, but there are signs of it appearing even in children in lower primary school.³⁶ Unfortunately, while there is some promising research, there is a limited understanding of how to reduce it.³⁷ Gaining an awareness of, and ability to recognise, the problem is the first step. Teachers should look out for students avoiding maths or displaying signs of anxiety ('freezing', sweating, fidgeting) when using maths, and use their knowledge of their students, and professional judgement, to support them to overcome their anxiety.

6

Use tasks and resources
to challenge and support
students' mathematics





Tasks are critical to the learning of maths because the tasks used in the classroom largely define what happens there. However, the evidence suggests that the *choice* of one particular task or resource over another is less important than the way that teachers set about using them in the classroom.³⁸ Tasks and resources are tools which need to be deployed effectively to have a positive impact on learning.

Effective use of tasks and resources requires a considerable level of skill: many teachers will require focused support to achieve this. School leaders should make this a priority for PL.

Using tasks effectively

What does the effective use of a task look like? The table below provides some key considerations with examples of what they mean in practice.

Use assessment of students' strengths and weaknesses to inform selection and use of tasks

A teacher asked students which of the following were equations of straight lines passing through the point (1, 2):

$$x = 1 \quad 5x = 3 + y \quad x = y - 1 \quad y = 2x^2$$

Some students checked to see whether (1, 2) satisfied these equations, but did not check that the equations were straight lines. Others dismissed all of the equations as they were not written in the form $y = mx + c$. This allowed the teacher to identify what students knew, and did not know, about the equations of straight-line graphs.

After discussing the answers and using graphical software to show the graphs, the teacher asked the students to create some examples and non-examples of their own for straight lines passing through the point (2, -3). They now produced correct examples written in various forms, such as $3x + 2y = 0$, and their non-examples included both lines that did not pass through (2, -3) as well as curves.

Evidence summary

The review was not able to identify meta-analyses on the use of tasks, although there is a great deal of literature on the use and design of tasks. One large-scale and robust study indicated that teacher knowledge is key to realising the learning potential of a task.

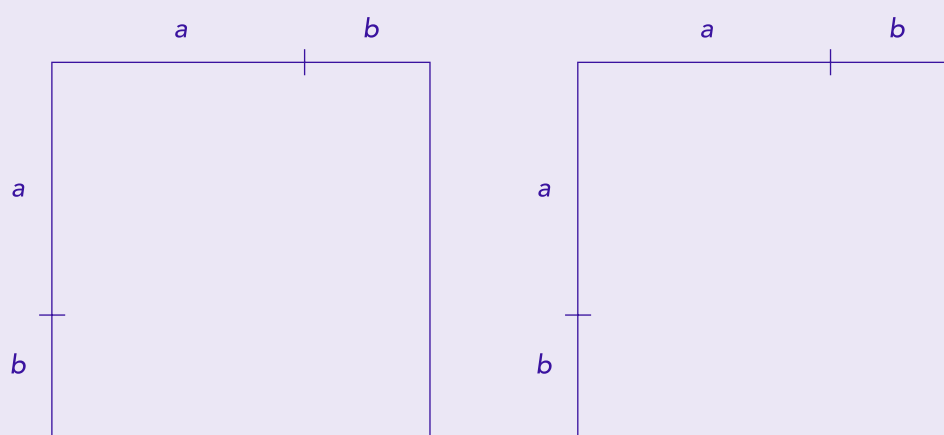
The review identified two relevant meta-analyses concerned with the effects of different textbooks. These provide moderate evidence indicating that the effects of one textbook scheme over another is at best small.

The review identified 11 meta-analyses addressing aspects of technology. Despite the large number of reviews, the evidence regarding technology is limited.



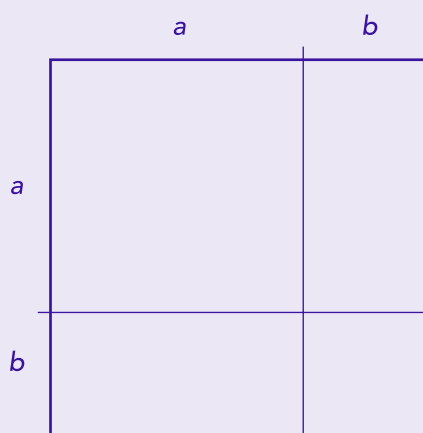
Use tasks to address students' misconceptions

The teacher noticed that some students seemed to assume that $a^2 + b^2 = (a + b)^2$. She drew two identical diagrams on the board.



She asked the students to represent $a^2 + b^2$ on one diagram and $(a + b)^2$ on the other.

By drawing two additional lines, as shown below, students could see that in general $a^2 + b^2 + 2ab = (a + b)^2$.

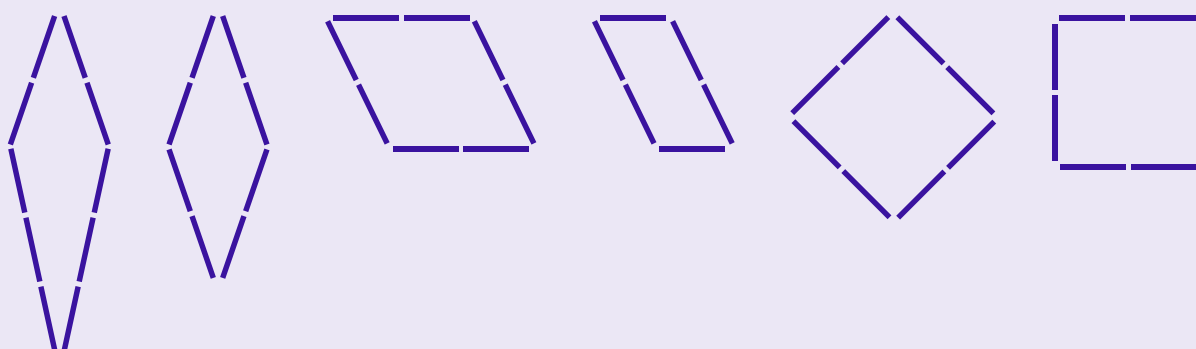




Provide examples and non-examples of concepts

A teacher asked for some definitions of 'rhombus', which she noted on the board.

She then revealed these shapes on the board, one at a time, each time asking, 'Put your hand up if you think it's a rhombus'.



She wrote the number of votes underneath each shape.

She then told the class that the first shape was not a rhombus but the second was. She asked for new votes for the remaining shapes, which she again recorded.

She then gave the answer for the third and fourth shapes and asked for new votes for the remaining two shapes, which she again recorded.

She then gave the answer for the last two shapes and asked once more for some definitions of a rhombus, which the class then discussed.

Here, the examples and non-examples were carefully chosen, in terms of both shape and orientation, in order to highlight common misunderstandings, such as that a square is not a rhombus.



Discuss and compare different solution approaches

A teacher asked a class to come up with different ways of calculating 5×18 .

Here are some of their approaches:

'I can multiply 5 by 20, then take two 5s away':
 $5 \times 18 = 5 \times 20 - 5 \times 2 = 100 - 10 = 90$

'To multiply by 5, it's easy. I can multiply by 10 then halve the answer.'
 $10 \times 18 = 180, \quad 180 \div 2 = 90$

'18 is 9 times 2, so I can multiply 5 by 9, then multiply the answer by 2.'
 $5 \times 9 = 45, \quad 45 \times 2 = 90$

The class discussed what similarities there were, how easy each method was to understand, and how efficient it was to execute mentally. The teacher asked the class to try to find similar ways to calculate 12×15 .

Use tasks to build conceptual knowledge in tandem with procedural knowledge

A teacher asked a class to perform this calculation using long multiplication:

$$\begin{array}{r} 34 \\ \times 52 \\ \hline \end{array}$$

The teacher then asked the students to move the digits around to produce a new 2-digit by 2-digit multiplication, for example 53×24 . She asked them to find the arrangement that gives the largest answer.

This task provided opportunities for students to rehearse the long multiplication algorithm (procedural knowledge) while at the same time developing conceptual understanding of place value.

Use stories and problems to help students understand maths

A teacher presented a class with this task:

1127 divided by 23 is equal to 49.

What number divided by 24 is equal to 49?

Some students noticed that 24 is 1 more than 23, and so gave the answer 1128.

Some used trial and improvement.

Some calculated 24×49 , which gave the correct answer, but not everyone understood why.

To help provide some insight into the structure of the task, the teacher read out the story below and then asked, 'how could we use this to help us solve the task?'

A class of 23 children earns \$1,127 for clearing litter from a beach.

They share the money equally and find that they get \$49 each.

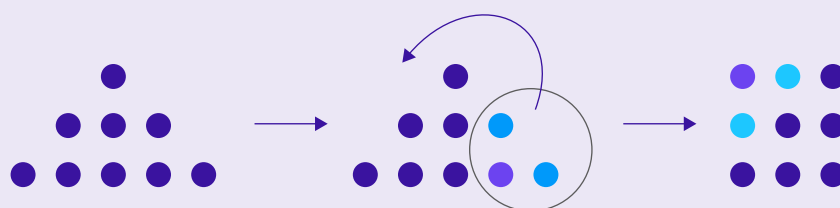
Students realised that they needed to pose a question like: 'How much money would a class of 24 children need to earn, so that when they shared it out equally they also each got \$49?'

In this context, the students found it easier to see that an additional \$49 would be needed for the 24th child, meaning that the answer would be $1,127 + 49 = 1,176$. Here the context illuminated the mathematical structure.

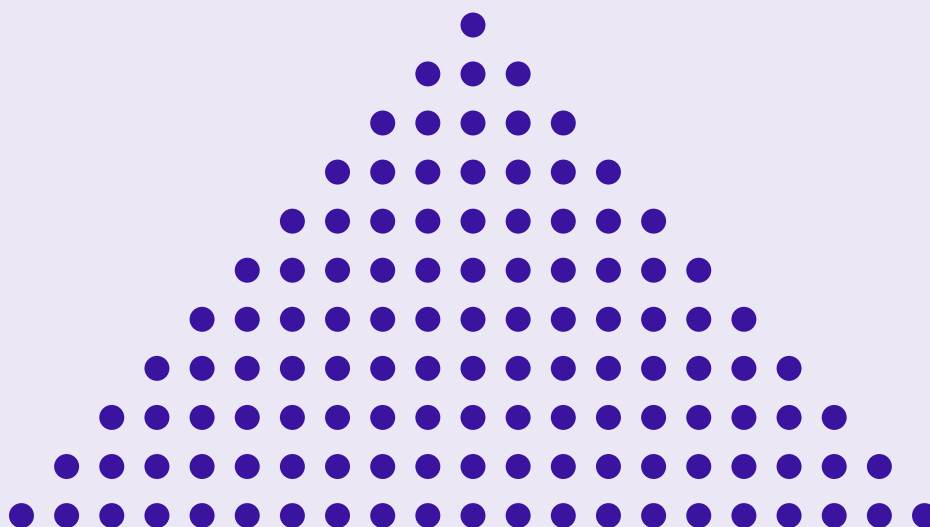


Provide opportunities for students to investigate mathematical structure and make generalisations

A teacher used the diagram below to show how a 'dot-triangle' with a base of 5 dots could be changed into a square array of 3×3 dots, just by moving 3 of the dots.



She then asked students how they could *calculate* (not count) the number of dots in a 'dot-triangle' with a base of 21 dots.



Some students just counted all of the dots.

Some thought that the triangle could be changed into a 19×19 array (as $3 = 5 - 2$ and $19 = 21 - 2$).

Others noticed that the dots could be rearranged into an 11×11 array (or, more generally, an $(n + 1) \times (n + 1)$ array for a base of $2n + 1$ dots).

These 'dot triangles' were used by the teacher to help students see that the sum of the odd numbers is a square number.



There are many potential sources for useful tasks. Tasks will often need preparation and adaptation to serve these purposes. It is likely that many tasks, even seemingly routine ones, can be used by a skilful teacher to support students to learn.

Using resources effectively

It is unlikely that introducing a resource on its own, whether it is a textbook or a new technology, will (on its own) have a positive impact on teaching or learning. Resources must support, or at least be accompanied by, an improvement in the quality of teaching to make a real difference.

Technology is a resource that appears to hold great promise for maths teaching, but the reality of its impact in the classroom has not always matched expectations. A great range of technological hardware and software is used in maths classrooms, including mobile devices, dynamic geometry software, exploratory computer environments, and educational games. Whatever the type of technology used, the evidence suggests a set of core principles for using it effectively. Here are three key considerations:³⁹

- 1. Identify a clear role for the technology in your students' learning.** Ask questions such as: What teaching strategies and tasks will help students explore the relationship between equations and graphs using dynamic geometry software? Will spreadsheets allow students to view and transform data more effectively? How can tasks be designed to harness the power of the technology itself to provide feedback to students?
- 2. Training for teachers should not just focus on the technological skills involved in using new equipment. Ongoing professional development on how the technology can be used to improve teaching** is likely to be needed if it is going to make a difference.
- 3. Before adopting a technology, consider the potential costs, including the impact on teachers' workload.** Added up, these costs can be greater than for similarly effective approaches which do not involve technology.

Evidence for Learning will publish guidance on *Using digital technology to improve learning*, focusing on the effective use of technology, in 2020.



7

Use structured
interventions to provide
additional support





Schools should focus on improvements to core classroom teaching that support all children in the class. With this in place, the need for catch-up intervention should decrease. Nevertheless, some high-quality, structured intervention may still be required for some students to make progress. Selection of the intervention should be guided by effective assessment of students' individual strengths and weaknesses.

Evidence summary

The guidance in this section draws on an EEF-funded review of maths interventions conducted by Ann Dowker.

The easiest way to identify high-quality interventions is to look for those that have been rigorously evaluated and have had a positive impact on student outcomes. However, few evaluations of maths catch-up interventions have been conducted, and an intervention with a rigorous and positive evaluation might not be available. Schools may want to adopt and implement an intervention with the features common to successful interventions:⁴⁰

- Interventions **should happen early**, both because mathematical difficulties can affect performance in other areas of the curriculum, and in order to reduce the risk of children developing negative attitudes and anxiety about maths.
- The intervention should be informed by the **evidence base regarding effective teaching and the typical development of mathematical capabilities**. How does the intervention relate to the recommendations in this Guidance Report?





- Interventions should include **explicit and systematic teaching**. This should include providing models of proficient problem solving, verbalisation of thought processes, guided practice, corrective feedback, and frequent cumulative review.¹⁷
- Effective implementation is essential to success. **Interventions require careful planning and use of school resources, including staff.** The best-designed program will not work if teaching staff are unavailable, excessively overburdened, or not adequately trained to deliver the program. Evidence for Learning's publication *Making Best Use of Teaching Assistants* provides guidance on the effective deployment of Teaching Assistants.
- Ensure that **connections are made between intervention and whole-class instruction.** Interventions are often quite separate from classroom activities. The key is to ensure that learning in interventions is consistent with, and extends, work inside the classroom and that students understand the links between them. It should not be assumed that students can consistently identify and make sense of these links on their own.
- **The intervention should motivate students** and prevent or counteract the association of maths with boredom or anxiety. The use of games, for example, is a recurring feature of promising programs, especially with primary school children.
- **Pay careful attention to what a student might miss if they take part in an intervention.** Will they be missing activities they enjoy? Will they miss out on curriculum content that they need to learn? Would class teaching be more effective? It is essential that the intervention is more effective than the instruction students would otherwise receive; if this is not the case, intervention students may fall further behind their peers.
- It is important to **avoid 'intervention fatigue'** from the perspective of both teachers and students. Interventions do not always need to be very time-consuming or intensive to be effective.

Box H: Specific learning difficulties in maths

There are major debates about whether there is a specific disorder that can be termed 'dyscalculia', whether it should be regarded as the lower end of a continuum of mathematical capabilities, or whether it occurs when students struggle with maths despite otherwise typical academic performance. We still do not yet know whether children with more severe or more specific mathematical difficulties require fundamentally different types of interventions from others.^{41, 42} If students are really struggling with maths, the most effective response is likely to be to attain a good understanding of their strengths and weaknesses, and target support accordingly (see Recommendation 1).

8

Support students to make
a successful transition
between primary and
secondary school





There is a large dip in mathematical attainment and attitudes towards maths as children move from primary to secondary school in Australia. For example, one of the largest randomised controlled trials in Australia commissioned by E4L tested a PL program that supports collaboration between Years 6–9 maths teachers. Teachers foster effective pedagogical content and continuity of learning strategies for students. The evaluation found that students whose teachers received the PL program made, on average, one month's additional progress in maths, however there were critical differences between year levels. Primary students made an additional two months progress in maths, which is promising, but there were two fewer months progress for secondary students.⁴³

It is clear that schools should be concerned about supporting students to make an effective transition. Unfortunately, there is very little evidence concerning the effectiveness of particular interventions that specifically address this dip. The broader research evidence, however, does suggest some key considerations:

- **Are primary and secondary schools developing a shared understanding of curriculum, teaching, and learning?** Both primary and secondary teachers are likely to be more effective if they are familiar with the maths curriculum and teaching methods outside of their age-phase.
- **How are primary schools ensuring that students leave with secure mathematical knowledge and understanding?** Primary schools could provide students with an effective defence against the common problems of transitioning to a new school.
- **When students arrive in Year 7, are secondary teachers attaining a good understanding of their strengths and weaknesses?** (See Recommendation 1) Do they use this information to build on key aspects of the primary maths curriculum in ways that are engaging, relevant and not simply repetitive?

- **How are secondary schools providing structured intervention support for Year 7 students** who are struggling to make progress (see Recommendation 7)?
- **How are students allocated to maths classes when they enter Year 7?** The research evidence suggests that allocating students to maths classes based on their prior attainment (often called 'setting' or 'ability grouping') does not, on average, lead to an increase in attainment overall and may widen attainment gaps. It has a slightly negative impact on students allocated to lower sets, although students allocated to higher sets may benefit slightly. Disadvantaged students are more likely to be assigned to lower sets, so setting is likely to lead to a widening of the attainment gap between disadvantaged students and their peers.⁴⁴

Evidence summary

There is little evidence concerning the effectiveness of particular interventions that specifically address the transition. This recommendation considers the broader evidence regarding effective teaching, and applies it to the specific problem posed by the transition. There are a large number of meta-analyses on setting, which consistently suggest it widens the gap between students allocated to lower and higher sets.

Acting on the evidence

Our learning is drawn from our experience and that of our UK partner, the EEF. Their work with schools suggests that making changes are not straight forward. Many of the recommendations speak to practical changes which can occur at the classroom level, while others require a more structural approach, such as the use of structured interventions (see [Recommendation 7](#)) and supporting transitions from primary to secondary schools (see [Recommendation 8](#)).

Evidence for Learning has produced a Guidance Report [*Putting evidence to work: a school's guide to implementation*](#) which can be used as a guide as you plan to implement changes. [Figure 1](#) provides an overview of the implementation process which schools can apply to any implementation challenge.



The stages of implementation

Foundations for good implementation

- ✓ Treat implementation as a process, not an event. Plan and execute it in stages.
- ✓ Create a leadership environment and school climate that is conducive to good implementation.

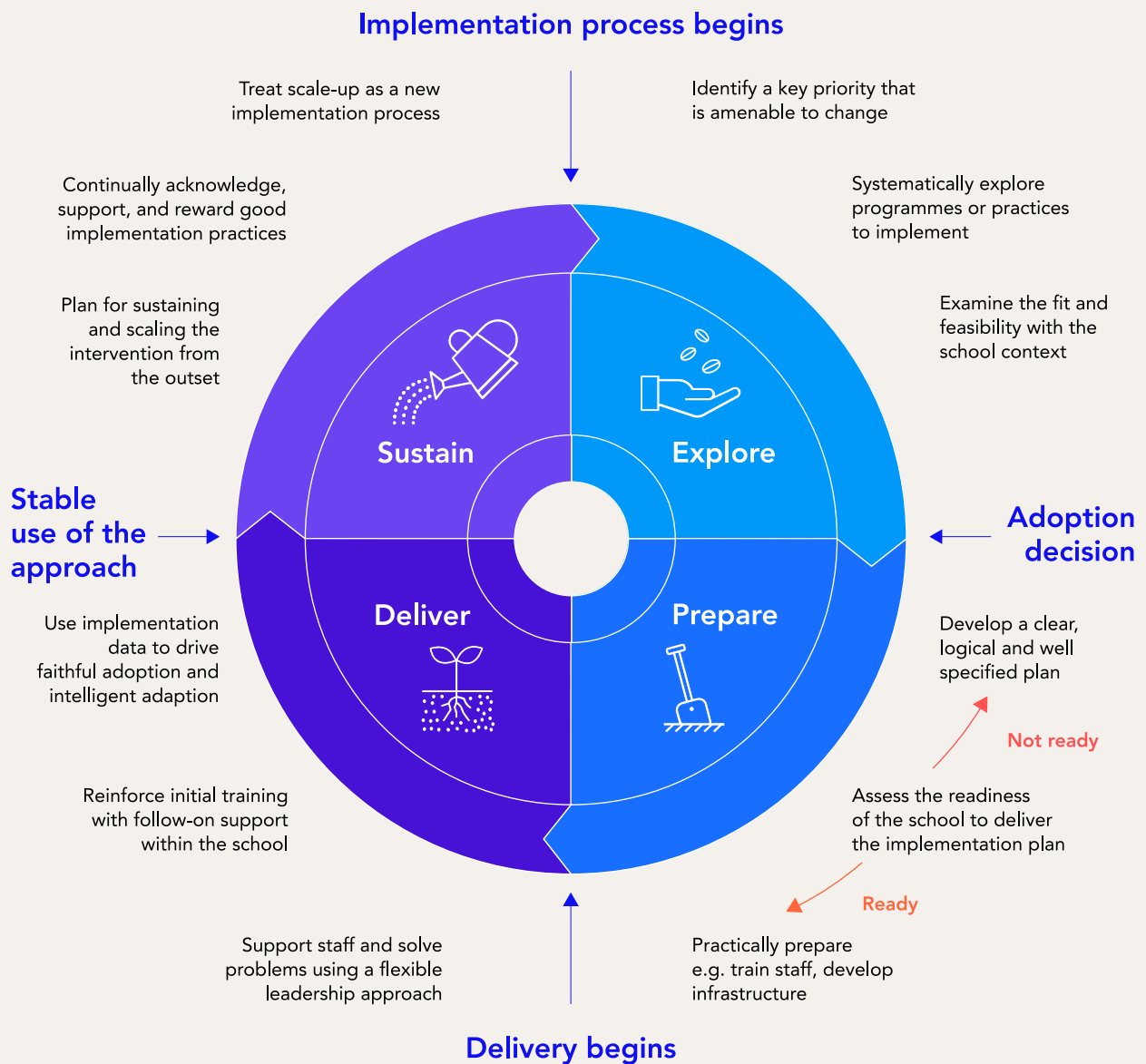


Figure 1: Implementation can be described as a series of stages relating to thinking about, preparing for, delivering, and sustaining change.

Further reading

The Australian Curriculum: Mathematics aims to ensure that 'students are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and work lives as active citizens.' The website contains resources for school leaders and teachers that will assist in the implementation of the recommendations outlined in the Guidance Report.

australiancurriculum.edu.au/f-10-curriculum/mathematics/

Evidence for Learning has produced a Red Amber Green (RAG) self-assessment guide which accompanies this report.

evidenceforlearning.org.au/guidance-reports/improving-mathematics-in-upper-primary-and-lower-secondary

Evidence for Learning highlights the international research available on several approaches which are relevant to this Guidance Report, including metacognition and self-regulation, and teaching assistants, within the Teaching & Learning Toolkit:

evidenceforlearning.org.au/the-toolkits/the-teaching-and-learning-toolkit/full-toolkit/

How was this guide compiled?

This Guidance Report draws on the best available evidence regarding the teaching of maths in upper primary and lower secondary school. The primary source of evidence for the recommendations is an evidence review conducted by Prof. Jeremy Hodgen, Dr Colin Foster, and Dr Rachel Marks.

The Guidance Report was created by our UK partners, the EEF, over three stages.

- 1. Scoping.** The process began with a consultation with teachers, academics, and other experts. The EEF team selected the area of interest (maths at upper primary and lower secondary school), appointed an Advisory Panel and evidence review team, and agreed research questions for the evidence review. The Advisory Panel consisted of both expert teachers and academics.
- 2. Evidence review.** The evidence review team conducted searches for the best available international evidence. Where possible, the review focused on meta-analyses and systematic reviews.
- 3. Writing recommendations.** The EEF worked with the support of the Advisory Panel to draft the recommendations. Academic and teaching experts were consulted on drafts of the report.

Glossary

Manipulatives	A physical object that students or teachers can touch and move, used to support the teaching and learning of maths. Popular manipulatives include Cuisenaire rods and Base Ten blocks or MAB blocks.
Meta-analysis	A particular type of systematic research review which focuses on the quantitative evidence from different studies and combines these statistically to seek a more reliable or more robust conclusion than can be drawn from separate studies.
Multiple	For any integers a and b , a is a multiple of b if a third integer c exists so that $a = bc$. For example, 14, 49 and 70 are all multiples of 7 because $14 = 7 \times 2$, $49 = 7 \times 7$ and $70 = 7 \times 10$; -21 is also a multiple of 7 since $-21 = 7 \times -3$. ¹²
Mixed fraction	A whole number and a fractional part expressed as a common fraction. Example: $1\frac{1}{3}$ is a mixed fraction. ¹²
Non-example	Something that is not an example of a concept.
Number bonds	A pair of numbers with a particular total, for example number bonds for ten are all pairs of whole numbers with the total 10. ¹²
Proportion	<ol style="list-style-type: none"> 1. A part-to-whole comparison. Example: where \$20 is shared between two people in the ratio 3:5, the first receives \$7.50 which is $\frac{3}{8}$ of the whole \$20. This is his proportion of the whole. 2. If two variables x and y are related by an equation of the form $y = kx$, then y is directly proportional to x; it may also be said that y varies directly as x. When y is plotted against x this produces a straight line graph through the origin. 3. If two variables x and y are related by an equation of the form $xy = k$, or equivalently $y = k/x$, where k is a constant and $x \neq 0$, $y \neq 0$, they vary in inverse proportion to each other.¹²
Rate	A measure of how quickly one quantity changes in comparison to another quantity. For example, speed is a measure of how distance travelled changes with time. ¹²
Ratio	A part to part comparison. The ratio of a to b is usually written $a:b$. For example, in a recipe for pastry fat and flour are mixed in the ratio 1:2 which means that the fat used has half the mass of the flour, i.e. amount of fat/amount of flour = $\frac{1}{2}$. Thus ratios are equivalent to particular fractional parts. ¹²
Representations	<p>'Representation' refers to a particular form in which maths is presented.¹² Examples of different representations include:</p> <ul style="list-style-type: none"> • two fractions could be represented on a number line; • a quadratic function could be expressed algebraically or presented visually as a graph; and • a probability distribution could be presented in a table or represented as a histogram.
Systematic review	A synthesis of the research evidence on a particular topic, that uses strict criteria to exclude studies that do not fit certain methodological requirements. Systematic reviews that provide a quantitative estimate of an effect size are called meta-analyses.

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Photo credentials:

- p3: Grange Primary School, part of the Learning Impact Fund Thinking Maths trial
- p5: Grange Primary School, part of the Learning Impact Fund Thinking Maths trial
- p10: Casimir Catholic College, part of the Learning Impact Fund QuickSmart Numeracy trial
- p30: St Felix Catholic Primary School, part of the Learning Impact Fund QuickSmart Numeracy trial
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